Notes on some Experiments with Built-up Columns.

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We all know that if we compare two bars of iron, of the same cross-section, both acting as columns, and, if one is very much longer than the other, the longer one will generally fail under a smaller load than the shorter, and the long piece may be expected to fail by bending sideways, while the shorter will crush or bulge. The ratio of length to least radius of gyration of section, at which the change in the mode of failure occurs, is not definite; it is different for different materials, and varies even for what is apparently the same material.

Increasing the transverse dimensions of a long solid bar, with a view to increasing the radius of gyration of its section, and so lessening the chance of lateral flexure, is an extravagant mode of procedure.

A better plan is to employ a hollow column, and thus increase the radius of gyration without unduly increasing the weight. But there is a limit to this, for, if we make the diameter of the hollow column very large and make the sides very thin, we may have failure by crumpling of the thin sheet of metal, i.e., secondary flexure,—where the longitudinal strips of metal into which we may imagine the tube to be divided, fail much as long columns would.

If, on the other hand, we employ metal in the sides of a tube, thick enough to avoid this crumpling or secondary flexure, we may again have extravagance. Such are the considerations that have led engineers to employ "built-up" columns. In these, instead of having very thin sheets, forming continuous sides, the material is gathered into a number of longitudinal bars, of L, or other suitable section, and these are laced together in such a way as to make them support each other. Without such lacing, each longitudinal would tend to yield by lateral flexure.

The object of the lacing in built-up columns is, therefore, to enable the several longitudinal components to render support to each other. If this is borne in mind, it will be seen that the view of the function of the lacing, which some persons hold, is not correct. This view is that the lacing should be designed to prevent deflection of the column as a whole, under an assumed eccentricity of loading, or under an assumed variation
in the modulus of elasticity on opposite sides of the column. The lacing is intended to make the components act together.

But there may be secondary flexure in the case of a built-up column as well as in the case of a thin tube. If the longitudinal components are too slight in proportion to the distances between the points where the lacing affords them support, they may fail by bending between these points. For this reason, it is advisable to make the ratio of length between lacing points to least radius of gyration of a cross section of a longitudinal the same as the ratio of length to least radius of gyration of the whole column.

And, further than this, seeing that, if the same quality of metal is employed throughout, it is desirable that the stress per square inch in compression shall be the same in all members, and seeing, further, as experiments show, that any piece of the lacing may be called upon to endure compression, it would appear to be advisable to retain the same ratio of length to radius of gyration for the lacing bars as for the whole column and for its longitudinal components. Existing practice is, however, very far from following this rule, as a glance at the thin flat bars, commonly employed to lace together heavy angles or channel bars, will show.

The elaborate experiments of Talbot and Moore, and of others, make it appear to be hopeless to determine by calculation what stresses the lacing bars may be subjected to. Very small defects in workmanship or material produce great effects in the stresses in the lacing bars. The experimental method appears to be called for in this case.

The object of making the experiments, referred to in these notes, was to ascertain the possibility, or otherwise, of determining, by a series of tests, suitable proportions and spacing for the side-lacing of columns of a certain type. The aim, in the first place, was to ascertain if possible how thick and wide lacing pieces must be if they are not to fail before the longitudinal members fail, and, in the second place, to find how closely the lacing points, i.e., the points of connection between longitudinals and lacing bars, must be spaced, if the longitudinal members are not to fail by flexure between the said lacing points—i.e., by secondary flexure.

The workmanship in the models which were tested may be taken to be of the same character as ordinary girder work,—if anything, a little inferior; but the fairly consistent results obtained show that there cannot have been very much wrong with the building of the columns. Only one of the 9 tests need be discarded.

During the application of load, each column was closely watched by several persons, with the object of detecting any local yielding, such as might result from imperfect workman-
ship; but, as usual in such cases, none was revealed; all failed so suddenly that it was impossible to say which part was the first to give way. Undulations moved along some of the longitudinals shortly before collapse, and the stress in certain of the lacing members changed sign; for example, it was observed that a piece of the lacing buckled, as in compression, at one stage, and straightened out tightly afterwards. But the collapse occurred with suddenness in each case. Inferences may, however, be drawn as to the order in which failure occurred in the various parts.

The columns experimented with were all 48 inches long by 4.8 inches square, this transverse dimension being measured outside the four longitudinal L bars placed at the corners. The material was mild steel; that in the longitudinals showed a not very well defined yield point at 38,000 lbs. per square inch, and the specimen tested, one inch in length, failed by the L opening out when the stress was 57,000 lbs. per square inch. The sectional areas of the L bars were ascertained by calculation from the weights of known lengths, taking 4.533 ounces as the weight of a cubic inch. The area of cross section of the L bar which was used in the longitudinals of all the columns, and in the flanges at their ends, was found in this way to be 0.145 sq. inches, so that the four longitudinal L bars, which supported the load, had a joint area of 0.58 sq. inches. The rivets were of diameters roughly proportioned to those that would be used in real girder work of similar design. Those used for attaching the flanges at the ends of the columns were 3-16 inch diameter. This size was also employed for attaching the lacing members to the longitudinals in columns 5 and 6. The lacing in columns 2, 3, 4, 2A, 3A, and 4A, was attached with rivets 0.12 inch in diameter, and in column No. 1, the diameter was 0.08 inch.

All the lacing pieces were steel L bars. As will be seen from the photographs, the columns numbered 1 to 6 had a single system of lacing on each of their sides, each piece making an angle of about 45 degrees with the longitudinals. One rivet at each end formed the attachment for each piece of lacing to the longitudinals. Naturally those lacing bars which failed in tension tore at the rivet holes; but those that failed in compression did so, in most cases, by deflecting laterally before damage was done at the riveted ends. The fact that so many lacing L bars failed in compression shows that flat bars are quite unsuited for lacing columns. The lacing was lightest in No. 1 and heaviest in No. 6, the variation in Nos. 2, 3, 4 and 5 being gradual.

The photographs taken of Column No. 1 after failure, show how the lacing was practically all destroyed, being unable to hold the L longitudinals together. Two of the longitudinal bars, diagonally opposite each other, bulged apart, while the
other two became bowed towards each other. The load at
collapse was 9150 lbs.

In column No. 2, the lacing failed mainly in compression,
and the longitudinals became bowed, but not to the same ex-
tent as in No. 1. This column failed at 17,320 lbs.

Inspection of table A makes it clear that neither 1 nor 2 were
sufficiently laced to enable the full strength of the longitudinals
to be developed. By strength of longitudinals in this case is
meant the strength of those portions of them between lacing
points, for, as will be seen, the lacing points in the first 6 col-
umns were not close enough to each other to prevent secondary
flexure. The ratio of length to least radius of gyration was
smaller for each entire column than for each section of a longi-
tudinal between lacing points.

Columns Nos. 3 to 6 all failed by secondary flexure at about
19,000 lbs., nothing being gained by having the lacing in 4, 5
and 6 heavier than in No. 3. In other words, the lacing bars
in No. 3, which had an area equivalent to 18.4% of that in the
longitudinals, were sufficiently strong to develop the strength
of the longitudinals when these had lacing points about 9½
inches apart.

Having got these results, which, under the circumstances,
were as satisfactory and consistent as could have been expected,
the next step taken was to try the effect of placing extra pieces
of lacing in columns resembling Nos. 2, 3 and 4, so as to re-
duce the distance between lacing points. The new columns
were numbered 2A, 3A and 4A. Their lacing bars had about
the same weight as those in 2, 3, and 4, but extra pieces were
placed transversely, extending from each apex in the former
system of lacing to midway between the lacing points on the
opposite longitudinal bar. The effect of introducing these ex-
tra pieces was to prevent secondary flexure in the longitudinals.

Column 2A was 17.1% stronger than 2.
Column 3A was less than 1% stronger than 3.
Column 4A was 21.3% stronger than 4.

The added metal in each case was 14.7% of that previously
there, in the whole column. The result in the case of column
3A may evidently be discarded, because some defect, which ob-
servation failed to discern, may have existed; or, as subse-
quent investigation made to appear probable, the lacing bars
in 3A may possibly have been 6% lighter than those in 3. The
writer has not been able to confirm this.

The result, then, of the three last tests is to show that the
system of lacing adopted in them is sufficient for developing
the strength of the longitudinal portions of the columns, for
each of the columns failed by primary flexure, i.e., they bent
sideways, each as a whole, without secondary flexure. So that
the experiments, few in number and small in scale as they are,
servc to demonstrate that, with the particular longitudinals
here made use of and with these dimensions for the columns, L lacing bars, whose weight per yard is about 18 or 20% of that of the longitudinal bars, and arranged as in these columns, are able to prevent secondary flexure, and are strong enough to hold the parts of the column together.

Further experiments, on a different scale, would be needed to show whether or not similar proportions would hold in the case of square, built-up columns with other sizes of longitudinal angle bars, or with different proportions of length to width.

It is true that some of the lacing members in the last 3 columns were broken or crumpled in the testing machine; but this may be taken to be the result—not the cause—of the collapse. We may take it that the column bent as a whole first—failing virtually—and then when the lateral deflection reached a certain amount, the lacing yielded.

This seems a reasonable inference, because these 3 columns 2A, 3A, and 4A failed in a manner quite different from 2, 3 and 4.

In the case of 3A, as mentioned, there probably was some defect, causing the premature yielding which there occurred.

The table of results (A) shows that the ultimate stress in the longitudinal L bars in column 4A was 38,276 lbs. per square inch, which is approximately the yield point of the mild steel from which the bars were rolled. It is not likely that any considerably higher figure for the ultimate strength of a column can be obtained with these longitudinal L bars, whatever system, or dimensions, of lacing be adopted, for the yield point in compression tests of this nature is virtually the ultimate strength.

It may be that columns, with longitudinal members, more massive in proportion to the transverse dimensions of the column than those experimented with here, might have their strength developed with lacing bars lighter in proportion than those in the present case; but it must be remembered that the question is often one of stability. Many columns have channels instead of L bars. The Quebec bridge had 4 channels as longitudinal members in the strut which caused failure. The lacing bars there had a section 0.32% of that of the column itself—a vastly lower figure than the 4.5 or 5% which we found necessary here. The Quebec bridge lacing, moreover, consisted of a double system of braces, connected by transverse pieces. The existence of these transverse pieces, which acted in tension, really weakened the structure, because they compelled the diagonals to share largely in the load on the strut which they were supposed merely to brace.

The writer is of opinion that systematic tests, comparing different sizes of lacing bars, and different systems of lacing, on columns of various sections, would afford much better guidance for the proportioning of struts than any formulae he has
seen. In any case, we should recognise that there is great uncertainty as to eccentricity of a column’s axis, or of loading, also that there are unknown variations in the strength and elasticity of the material of which a column is built, and, further, that accidental lateral blows may be given, which may be cumulative with considerable wind pressure; besides all which if the strut is not vertical, the column’s own weight tends to deflect it. The attachment of a cross beam in a bridge may deflect a strut. For all these reasons it is urged that lacing bars should be used liberally and not cut down for economy’s sake. Extra dimensions in the parts forming columns may be made to give great increase of strength, and the cost is comparatively small.

For several years past, and especially since the collapse of the Quebec bridge, scores of formulae for columns have been brought forward, and much excellent experimental work has been done. The researches of Lilley, Burr, Buchanan and of Talbot and Moore on steel struts has supplemented what Hodgkinson did with cast iron and Christie with wrought iron. Lilley’s work has thrown much light on the subject of secondary flexure, mainly in the case of hollow cylinders. Buchanan has given the results of actual tests to destruction of full-sized bridge struts. Burr tested a large model of the strut whose failure wrecked the Quebec bridge. Talbot and Moore’s researches are especially valuable, inasmuch as they show how unexpectedly and how seriously stresses vary from place to place in a column. They applied numerous extensometers to longitudinal and lacing members of actual bridge struts and deduced the corresponding stresses. A very important finding of theirs was that the actual stresses were very much greater than the measured deflections would account for. They ascertained that, in the particular columns which they tested, the maximum stresses in the lacing bars were such as would be produced by transverse loads varying from $2\%$ to $6\%$ of the central compressive load.

Painstaking work has been done by Moncrieff, who analysed the experiments of others in the endeavour to find what deflection it would be safe to assume for the calculation of bending moments and shearing forces.

The well known formulae of Rankine, Gordon and Euler, as well as the straight line formula, so popular in America, have been examined and criticised by numerous writers to the technical press. One writer points out that, if Rankine’s formula be put in a certain form, it will be found to be based upon a certain assumed eccentricity of loading, this eccentricity varying as $\frac{l^2}{b}$, where $l$ is the length of the column and $b$ its least transverse dimension. Again, if the straight line formula be looked at in a similar manner, it, also, will be found to be based
upon an assumed eccentricity, the eccentricity in this case, however, varying as the length of the column, simply.

Claxton Fidler bases his formula upon the assumption that the modulus of elasticity of the material on one side of a column differs from that on the other side by an amount equal to the greatest known difference in the values of the said modulus, in other words, he goes back and deals with the causes of the deflections which others assume.

Keelhoff derives a formula by calculating the bending moment and stress corresponding to an indefinitely small deflection.

A number of investigators appear to apply Euler's method of reasoning to determine a critical load, which corresponds to any deflection, and then proceed in a way which is tantamount to giving the said indefinite deflection a definite value.

In the majority of cases, the procedure followed in the preparation of a formula is to find a value for a bending moment and calculate the corresponding shear, which is, of course, a maximum at the ends and zero at the middle of the strut. Now, as Carl Jensen points out, lacing designed on these principles might prove dangerously weak, for the stress may be quite large at the middle of a column's length; for example, when the centres of pressure on the two ends of a column are on opposite sides of the column's geometrical axis, then the column will tend to fail by bending in an S curve, causing heavy shear at mid length. Burr's large model column failed in this manner. Instead, then, of basing our calculations upon some assumed deflection of a column as a whole, i.e., a bending such that the maximum deflection is at the middle, we should remember that it may bend in an S curve; and if, for purposes of approximation, we employ one of the many formulae, deduced as above-mentioned, we should certainly not reduce the dimensions of the lacing near the middle of the column, but maintain, throughout, the dimensions found for the lacing at the ends.

But, rather than trust to any formula or to any factor of safety—which may be guessed to be sufficiently large to cover unknown possibilities, the writer would recommend that those concerned with the design of important compression members should ascertain experimentally what would be safe dimensions to employ for the lacing components of those columns. The results of a series of such tests might be used to base designs upon in much the same way that the results of tests of timber columns are employed.

ADDENDA.

(1) All the L bars employed, for longitudinals, heads of columns and lacing, had sides of equal width. The columns were thus not unlike those used in poppet-heads for important mines.
(2) The ratio of length to least radius of gyration was, in the case of the complete column — = 22 to 1. The corresponding ratio for the portion of each longitudinal member of a column, contained between adjacent lacing points, was, for columns Nos. 1 to 6 about 55 to 1.

In columns 2A, 3A, and 4A, this ratio was 28 to 1.

The ratio in the case of the lacing bars was, in general, greater still. That for column 2A, for instance, was 78 to 1; and yet this lacing proved sufficient to develop the strength of the column as a whole, i.e., the column failed as a unit, without appreciable distortion such as would result from the yielding of parts.

The common rule—and a good one to follow—is to make the ratio of length to least radius of gyration, for each portion between lacing points of a longitudinal component of a column, the same as for the whole column. The present experiments support the rule, e.g., secondary flexure was observed when the ratio was about 55 to 1, and the lacing bars strong enough, while the column proved to be satisfactorily braced against secondary flexure when the lacing points were so spaced as to make the ratio 28 to 1.

On the other hand, it must be borne in mind that experiments, such as those carried out by Talbot and Moore, show that any lacing bar may act in compression. Now, if the same stress per unit of area be allowed in lacing bars as in the principal components of a column, it would follow that the ratio of length to least radius of gyration should be the same for lacing bars as for the other portions of the column. The practice is to make this ratio far greater for the lacing than for the principal bars, and the present experiments seem to show that the practice is to some extent warranted. This leads to

(3) The object of lacing. In the writer's opinion, the object is to hold the longitudinal members together and so prevent them from yielding singly, by lateral deflection, as long columns. This is not the same thing as providing against deflection of the column as a whole. It seems incorrect to design a column as a beam subjected to bending, because the column, under working loads, should not deflect as a whole, nor, with proper workmanship, should there be any perceptible initial deflection. Moreover, it is only in exceptional cases that loads are applied eccentrically. Nor is it sufficient to consider each longitudinal component separately, designing the lacing to resist bending in two planes, for it must be remembered that the lacing bars fixed to one longitudinal are merely attached to other longitudinals, which may tend to bend in the same direction as the first. This is not the same thing as staying an upright post from an immovable wall.
There appears to be no method of deducing a rational formula for determining the dimensions of lacing bars connecting longitudinal column components with each other: Their raison d'être is (1) to prevent flexure of individual longitudinal components, and, (2), to transmit any inequality of loading from one such component to the other. What appears to be needed is a series of tests, covering built-up columns of all good designs. Nothing but actual tests can enable us to make satisfactory allowance for the accidental imperfections in workmanship, the variation in elasticity of the materials used in construction, and unintentional eccentricity of loading.

(4) The failure of column No. 1 would suggest the usefulness, in certain cases, of bracing diagonally opposite longitudinal column components together.

(5) The columns tested were placed horizontally in the testing machine. It might, therefore, be expected that their own weights would cause slight initial downward bending, great enough to influence the direction in which final collapse would occur. But, curiously enough, none of the columns bent downwards at collapse: most of them bent upwards, and the remainder bent sideways.

(6) The following are a few quotations from the paper by Talbot and Moore, referred to in the foregoing notes. [University of Illinois Bulletin, No. 44, "An Investigation of Built-up Columns Under Load." Also Proc. Am. Soc. C.E., June, 1909.]

P. 63. "It seems futile to attempt to determine the stresses which may be expected in column lacing for central loading by analysis based on theoretical considerations, or on data now available."

"No relation has been found between the stresses actually observed and the stresses computed by column formulas. The stresses do not increase toward the middle of the length" [of the longitudinals] "of the column, as may be expected from the Rankine form of analysis, but are quite irregular in their location and distribution."

P. 61. "For the strength of the component angle, channel, or other structural shape used in a built-up compression piece, many engineers have been satisfied with the provision that the slenderness ratio," (i.e., the ratio, length: least radius of gyration) "of the component member shall be less for the length between the points of attachment of lacing than the slenderness ratio for the column as a whole, and have given little attention to the possible non-integrity of the section or to the probable effect of imperfections of manufacture. Fortunately the large influence of the slenderness ratio in column formulas has given sections with which failures have not occurred. Whether a column formula should include a factor depending on the form of the section and the relative thickness of the metal, or whether the allowable stresses for any form of column should
be based on experimental data for the section used, will depend on future developments."

P. 60. "Within the critical length at which Euler's formula governs, the general flexure of the column as a whole under load has less influence upon the strength of the column than is ordinarily assigned to it, and therefore the influence of \( \frac{1}{r} \) (length: least ratio of gyration), "is not as great as is represented in the usual column formula."
### TABLE A.

**Steel L Bars used for Longitudinal and End Flanges.**

Weighed, 23.635 oz. per yard and measured 0.6 in. x 0.6 in.; Section Area, 0.145 sq. in.

<table>
<thead>
<tr>
<th>Column No.</th>
<th>Dimensions of Lacing bars.</th>
<th>Weight of lacing bars per yard.</th>
<th>Section of lacing bar compared with Longitudinal bar.</th>
<th>Collapsing load.</th>
<th>Stress in Longitudinals at collapse of column.</th>
<th>Remarks on Appearance at Failure.</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>in, in, 7/32 x 7/32</td>
<td>oz. 1.25 % 5.28</td>
<td>lbs. 9,150</td>
<td>lbs. per sq. in. 15.776</td>
<td>Lacing nearly all destroyed; some torn at rivet holes, others buckled. Longitudinals bowed.</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>19/64 x 19/64</td>
<td>2.67 11.31</td>
<td>17,320</td>
<td>29,862</td>
<td>Lacing buckled; one piece torn at rivet hole. Longitudinals bowed.</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>13/32 x 13/32</td>
<td>4.36 18.40</td>
<td>19,110</td>
<td>32,948</td>
<td>2 diagonals torn at rivet holes; one buckled. Secondary flexure in longitudinals.</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>1/2 x 1/2</td>
<td>5.00 21.16</td>
<td>18,300</td>
<td>31,552</td>
<td>3 diagonals buckled. Secondary flexure in longitudinals.</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>5/8 x 5/8</td>
<td>6.60 27.90</td>
<td>19,500</td>
<td>33,621</td>
<td>No lacing damaged. Secondary flexure in longitudinals.</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>9/16 x 9/16</td>
<td>8.78 37.10</td>
<td>18,960</td>
<td>32,586</td>
<td>No buckling of lacing. One sheared rivet. Secondary flexure in longitudinals.</td>
<td></td>
</tr>
<tr>
<td>2A</td>
<td>1/4 x 1/4</td>
<td>2.67 11.31</td>
<td>20,280</td>
<td>34,966</td>
<td>Column bent bodily. Some diagonals buckled; 3 torn at rivet holes.</td>
<td></td>
</tr>
<tr>
<td>4A</td>
<td>15/32 x 15/32</td>
<td>5.00 21.16</td>
<td>22,290</td>
<td>38,276</td>
<td>Column bent bodily. Many diagonals torn and buckled.</td>
<td></td>
</tr>
</tbody>
</table>
Columns 1 to 6 before Testing.

Columns 1 to 6 after Testing. — Another view.

Columns 2A, 3A and 4A after Testing.

Columns 1 to 6 after Testing.

Columns 2A, 3A and 4A after Testing. — View at right angles to previous view.

Columns 2A, 3A and 4A after Testing.