MECHANICAL SOLUTION OF PROBLEMS IN ELECTRICAL NETWORKS.

By Roy J. Bennie.

Hitherto, problems of flow in complex electrical networks have been soluble practically only by constructing the networks, or scale models of them, and loading them with appropriate electric currents. Many useful resistance-inductance-capacity boards are now being constructed at great expense for the solution of such problems—especially those concerned with extra high tension circuits subject to electrical surges.

This paper describes a mechanical method whereby complex problems involving steady flow may be solved with accuracy sufficient for all practical purposes. The process is based upon the identity of terms of the equations expressing flow in conductors and those connecting the forces, tensions and deflections of funicular structures.

BASIC PRINCIPLES.

Figure (1) shows a string hung over two pulleys, A and B, and subject to tension $T$, due to the weights suspended at the ends. A weight $W$ is suspended from the cord at $C$, and an equal force is applied in an upward direction at $D$. One of the pulleys, $A$ or $B$, is raised or lowered until the strings $DA$ and $BC$ are horizontal. Then we derive the equation:

$$W : T = y : l \text{ or } W = yT/l \ldots \ldots \ldots (1)$$

Similarly, in an electrical circuit: if current $C$ enters a conductor and leaves it after passing along a length $l'$ of the conductor of resistance $1/k$ per unit length, if $E$ is the potential drop between the points of entrance and exit, then from Ohm’s law we have the equation $C = E k/l' \ldots \ldots \ldots (2)$

The equations (1) and (2) are identical in form. Therefore, it is clear that if a model in strings and weights be constructed analogous to a complex electrical network, so
that the terms of the equations of behaviour of its various members will be identical with the corresponding terms of the equations representing the electrical behaviour of the corresponding conductors, then it is possible to apply this complex funicular model to represent a network of electrical conductors, and to load it in such a manner as to determine the electrical behaviour of the various conductors under the incident electrical conditions.

Figures (2) (3) and (4) show the funicular arrangement for simple circuits. In Figure (2) the current \( C = w_1 + w_2 + w_3 + w_4 \) at \( A \) enters a conductor of uniform resistance, and portions are drawn from the wire at \( D, E, F \) and \( G \). It is observed that the gradient is steeper where the potential gradient is steeper. Figure (3) shows the treatment of a cord corresponding to a conductor with change of conductivity at \( A \). Figure (4) represents the analogue of two portions of a circuit, in parallel, fed from the same supply at \( A \). The two ends of the conductors, at \( A \) and at \( B \), are on the same level, representing that they are at the same potential, and therefore are equivalent to being connected to common terminals. More complex examples will be shown later.

In all these examples the following analogues are to be noted:

(a) Currents supplied to or drawn from conductors are represented by upward or downward thrusts respectively.

(b) Resistance-per-unit-length is represented by the reciprocal of the horizontal component of the tension of the string.

(c) Resistance of a given length is represented by the corresponding length in the model divided by the tension.
(d) Potential difference between any two parts of the network is represented by difference of altitude of the corresponding points in the model.

(e) Potential gradient is represented by gradient of the string; and, consequently, this may serve also as a measure of current density and as a mode of determining the current flowing in the conductor.

\[ T + \omega \frac{r \sin 45^\circ}{R + r \cos 45^\circ} \]

**Fig 8**

\[ T \]

**Fig 9**

\[ T, \frac{R + r \sin \alpha}{R + r \cos \alpha} + \omega \frac{r \sin \alpha}{R + r \cos \alpha} \]

**Fig 10**

**Fig 11**

**Fig 12**

**ERRORS AND THEIR PRACTICAL ELIMINATION.**

1. **Inflexibility of Cord Tends to Lessen Relative Deflections:** Fine threads may be used with light weights; alternatively, piano-wire with relatively heavy weights in order to eliminate the effects of rigidity. This error is then imperceptible.

2. **Friction of Pulleys:** Pulleys may be mounted on ball bearings or conical pivots. The author prefers to use pulleys, the axles of which roll on inclined planes set at 45 deg. to the horizontal as shown in Figure (6). The error due to rolling friction is imperceptible.

3. **Weight of Pulley and its Axle (Figure 8):** If \( r \) is the radius of the axle, \( R \) the radius of the pulley, and \( w \) the weight of pulley and axle, then the tension in the horizontal thread is greater than that of the weight \( T \) by

\[ \frac{wr \sin 45^\circ}{R + r \cos 45^\circ} \]

As the weight \( w \) is less than one-twentieth \( T \), and \( r: R \) is less than 1:10, the tension on the thread will not be enhanced
by more than about one part in three hundred. If desired, it may be compensated for by hanging a weight over the pulley as shown in Figure (9), or by reducing $T$.

4. Inclined Planes: Failure to Set at 45° (Figure 10): The lever arms of the two portions of the thread about the fulcrum will be in the ratio: $$(\frac{R + r \sin \alpha}{R + r \cos \alpha})$$

This error is not likely to be as large as that due to the weight of the pulley.

5. Terminals of Threads not Adjusted Absolutely Horizontal: When properly adjusted, all threads between the last flexure and the support or pulley must be horizontal. This may be assured in practice by providing a frame of sufficient size to enable those portions of the threads to be of length sufficient to secure accuracy in setting.

6. Inaccurate Location of Threads: The model must be carefully set out to correspond with the electrical analogue. When deflected under the loading, any necessary readjustments must be made; each thread must be finally in a vertical plane. Grades must not be allowed so steep as to cause cross threads to slide down the grade. If this occurs, a lighter scale of loads must be applied, or greater tensions employed, or the threads must be clipped together with a light clip.

7. Friction of Thread on Thread: Providing that each thread is in a vertical plane, there will be no error due to this cause.

It is clear then that the process is amenable to an accuracy equal to that of a graphical construction to the same scale.

**EXAMPLE OF APPLICATION.**

Suppose that it is desired to ascertain

(a) The potential drop on the track of an electric tram line of length 6,000 yds., of resistance one-tenth ohm per 1,000 yds., due to four trams, each delivering 30 amperes to the track:

(b) The effect of conveying the current to the power-station entirely by an insulated return negative feeder of twice the resistance of the track, which drains the track through six resistances spaced 1,000 yds. apart (see Figure 11).

Figure (5) shows a wooden frame (about 8 ft. long by 4 ft. broad) set on a perfectly level table. The side facing the observer is set at 45°, the opposite side is vertical. Battens are fixed to these faces so that small bars may be
slipped in between them and the frame, for the purpose of holding threads or supporting pulleys (see Figure 6).

A convenient scale of voltage in this problem is 1/6 inch = 1 volt; and we shall assume 10 inches represents 1,000 yds. Then: If 30 amperes in 1,000 yds. produces 3 volts D.P. and 1 ounce in 10 inches is required to produce 3/6 or 1/2 inch deflection, consequently, from Figure (1), the tension on the thread must be 20 ounces. Accordingly, a 20 oz. weight is hung on the end of the thread (Figure 5) and the four 1 oz. weights are hung wherever it is desired to represent the trams, and a 4 oz. weight is suspended from the thread going over the pulley above the frame, balancing the four 1 oz. weights, i.e., representing 4 x 30 = 120 amperes.

(a) The end of the thread is now slid up or down the rod at the end of the frame until both extremities are horizontal, as revealed by careful tests with a scribing block. Then the difference in altitude of the two ends is observed to be about 6 inches, corresponding to a potential drop of 36 volts. This solves the first part of the problem; it is a simple matter to calculate this.

(b) Since the feeder is to have resistance equal to one-fifth ohm per 1,000 yds., a thread is now stretched parallel to the “track” and 6 inches away therefrom, and a 10 oz. weight suspended from its end. The upward force of the 4 oz. weight is now transferred from the “track” to the “feeder,” corresponding to the new circuit (Figures 6 and 11). Then threads are carried from the vertical bars, in the back of the frame, over the “feeder” and under the “track” and then over pulleys in front of the frame, and weights are attached to their ends. The extremities of all the threads are adjusted horizontally with the aid of a scribing block, by raising or lowering the pulleys or the ends of the threads. Various experiments may be made with different tensions on the cross threads, and the most desirable track condition secured after a few trials, moving the “trams” and perhaps altering the “resistance” of the “feeder.” A greatly reduced potential difference is noted (Cf. Figures 5 and 6).

The resistance of any of the ties between the feeder and rail may be calculated thus: Example: One feeder tie is represented with a tension of 2 oz., and the horizontal distance between feeder and rail threads is 6 inches. Then, since 10 inches on a string loaded with 20 oz. represents 1/10 ohm, 6 inches on a string loaded with 2 oz. corresponds to a resistance of one and two-thirds ohms.

It is clear that the model need not represent a geometrical copy of its electrical counterpart.

Figure (7) shows the construction of a frame for solving problems in more complicated networks. Two faces are in-
clined at 45 deg. to the plane of the table and all four faces are fitted with battens which hold in place the bars supporting the strings and pulleys. Wherever the section of one of the conductors is changed, the tension of the thread is varied. This may be done as indicated in Figures (3) or (12). Where floor space is not available the apparatus can be arranged against a flat wall. It will determine the flow in any system of conductors, no matter how complex, where we are given the potentials or currents at all points of entry and exit. It is specially useful in determining the balance in three-wire direct current circuits.

This process may be applied to a very wide range of problems. Among other applications, the author applied it during an electrolysis survey, to determine the source of errant currents. A very light muslin cloth was lightly stretched on a frame like Figure (7) by numerous threads and pulleys, etc. Strong threads were sewn through it to represent the pipes and tram track. The attempt was made to locate a source of the current leakage by bringing the "rail" and adjacent "pipe" to the same potential at some point and checking the deduced currents against those observed in the survey. Lack of simultaneity of current readings in the survey prevented entire success. Here, of course, the equations of flow of the earth currents were not quite identical with the distortions of the web of cloth, which corresponds to flow in a lamina, as pointed out by A. A. Griffith and G. J. Taylor, in connection with soap films.*

Unfortunately, this process is not applicable to the solution of problems in the flow of fluids in pipes.

**Merits of the Process.**

1. Prior to commencing an extension, its effect on the existing work can be ascertained. Complete reticulation of new districts can be studied beforehand.

2. The whole system is simultaneously visible in the form of a three dimensional model. Thus the effect of any alteration is instantly observed throughout the whole system.

3. Accuracy depends solely upon accurate setting out.

4. The apparatus is cheap and can be very speedily applied.

5. If in a special case strict accuracy is essential, the approximate results obtained in this process may be inserted in the equations of flow, and the ultimate values obtained therefrom by successive approximations.

6. The apparatus may be used for solving problems of flow in heterogeneous laminae.

DISCUSSION.

The President moved a hearty vote of thanks to Mr. Bennie, and thought their thanks were especially due to him for the trouble he had taken in constructing the models. Mr. Bennie usually dealt with problems in an original fashion, and that characteristic was noticeable in his treatment of the problems enunciated in the lecture.

Mr. W. ISON seconded. About 20 years ago a problem arose in connection with a work in which he was interested—namely, the axle loads of a locomotive—which was solved in a somewhat similar manner. To obtain the upward forces they used spring balances, and the downward forces were secured by dead weights.

Mr. C. F. LINDBLADE said it had given him great pleasure to hear the paper. The demonstrations would appeal more to those connected with electrical work than to others. One model was especially interesting to him, as it represented practically a lay-out of the City of Melbourne. It was laid out in square sections. In the case of a city laid out like Melbourne it was almost impossible to calculate what potential drop would occur if the loading were shifted to different areas. But, with a model such as that exhibited, they could certainly determine what was happening. Even the drop along a feeder with a number of taps was not such an easy thing as sometimes appeared. The problem of electrolysis was a difficult matter. Everything might be all right over a given area, and then someone might put in a new water main or a new telephone cable; thus conductivity was introduced, and the original survey was totally upset. By the use of Mr. Bennie's model they could satisfy themselves with reasonable accuracy with what was likely to happen. Fortunately the inner city was fairly free from the troubles they had been speaking of. There were so many pipes that they were very slightly affected. It was the outer districts that were most seriously affected, where tramway systems existed and underground pipes were not very numerous. The first model was capable of giving very good and closely accurate results in determining flows. Of course, there were many electrical means of doing that, by means of the calculating boards which had been used in big power schemes for the determination of flow and power in networks. It was very difficult to know what happened in a big system when a short circuit occurred, unless one of these electrical boards was used. Ammeters could be inserted at different points, then a short circuit was used and the whole of the instruments read as quickly as possible to ascertain what power was flowing
through the different circuits. With the apparatus demonstrated, the thing was reduced to simplicity and was comparatively easy in its working.

Mr. W. Reid Bell said the apparatus had opened their eyes to the possibilities outside electrical matters. It was a most valuable contribution to scientific knowledge.

Mr. Wm. Chas. Rowe thanked Mr. Bennie for his paper. He had brought forward the point that by mechanics they should be able to solve their complex problems. He thought Mr. Bennie had placed before them matter which engineers would be able to use in other directions.

The President thought engineers had been too prone to rely entirely on mathematics to solve their problems, and had not paid sufficient attention to graphical methods of illustrating their problems. One of the great charms of the paper was the demonstration that in a comparatively simple manner they could solve difficulties that would tax the resources of a mathematician of considerable ability. He believed that in many cases higher mathematics were used when more simple methods would serve the purpose.

Mr. R. J. Bennie, in reply, said he was grateful for the warm reception accorded the paper. Its preparation had given him great pleasure. The evolution of methods for the solution of problems of complex nature, such as dealt with in the paper, possessed for him a great fascination. He had successfully applied the methods described to the solution of actual problems, and he hoped that others would find them equally useful. He thoroughly endorsed the remarks of the President and Mr. Rowe that present-day practice tended towards the mechanical solution of complex problems; or, rather, the solution, by analogy, of problems of complexity beyond ordinary calculation. These methods were all based upon the identity of terms of analogous equations. In a sense, one should have a mathematical mind to perceive these relations in the first place; but that did not imply that there was any difficulty in the application of the method when once evolved. Simple arithmetic generally sufficed. In evolving methods of this nature he had always attempted to preserve the active distinct from the passive elements, so that the one could be varied without a complete reconstruction of the whole apparatus. Here they had something which represented the passive network of conductors; the active principle, viz., applied currents, then produced an effect thereon. Either could be varied without a complete reconstruction of
EXHIBITS.

INCANDESCENT LAMPS.

Mr. C. F. LINDBLADE exhibited a number of electric lamps. He said the first lamps were of platinum wire, which had their troubles owing to the low temperature for safety of platinum imposing limitations. The next was the carbon filament, which at the time showed some improvement. About 15 years ago they saw the end of the carbon filament lamp. He exhibited one of the earliest lamps that came into Melbourne—one of the lamps used in the old Theatre Royal. It was a flat carbonised filament. He then showed a lamp which was produced some eight or nine years later, in which Edison and others found a difficulty that was known as the Edison effect, a discharge between the lead-in conductors. To overcome that, the legs were placed considerably farther apart. Then followed the carbon lamp, upon which there was very little improvement made for a number of years, until a metal filament lamp was introduced in which a special process was used to ethylise the filament. The next was the Tantalum lamp. It was a squirted filament, but the difficulty with that lamp was that two had to be used in series. Following that came the squirted tungsten lamp. Tungsten in the early days was only made as a black powder. That was mixed to a paste, and before being placed in the lamp was brought to a high temperature, when the small particles welded themselves together and became a filament. That was satisfactory until they had to be transported, when, owing to the brittle nature of the filament, they were more easily broken. But after some time a method was discovered of manufacturing tungsten by heating the tungsten powder in a gas furnace, and then hammering it until it assumed the form of a small rod. Then were shown some filaments from gas-filled lamps, in various shapes and colours, due to air getting

the model. In view of the rapid growth of instrumental means for the solution of engineering problems, he felt he should sound a note of warning: Where big issues were at stake, such problems should not be entrusted to persons untrained in instrumental manipulation. The study of instrumental error was a most important branch of physics. Instrumental calculation must be used with the greatest circumspection; and, wherever possible, the results should be checked in the equations.
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Title:
Mechanical solution of problems in electrical Networks (Paper & Discussion)

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Mechanical solution of problems in electrical Networks (Paper & Discussion)